Logic and the Dynamics of Subjective Truth

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Logic is the most fundamental framework of our understanding of the world in a formal language. The ability to say that a statement is true or false is basic to everything we know as mathematics. Despite its fundamental nature it is also difficult to understand fully.

Formal logic is often considered to be about how we infer the truth or falsehood of statements from other true or false statements. Given a set of assumed true and false statements, the question is what can we say about other statements. Inference is considered to be independent of the actual statements that are made and only depend on whether they are true or false.

A key purpose of the framework of logic is to understand how people can engage in discourse that enables them to jointly agree on truth and falsehood of statements in a way that precludes disagreement because of the underlying validity of the process of inference.

Thus, if A is true and B is true then we should be able to say "both A and B" are true without debate—and without regard for what A and B are actually saying—a victory for clarity and mutual agreement.

The purpose of this discussion is to clarify the challenges of applying logic to the real world. Logic is a framework that we wish to use to characterize the world, and our understanding of it. While what I say here is not new, I have not seen a discussion of logic that presents its limitations in describing the real world with great clarity. Philosophers and logicians have expanded upon and described mathematical and other limitations of logic in describing the world, but those discussions I have read are difficult to follow and opaque. What is left is that many consider logic to itself be true and the world should adjust to it, rather than that there is a need to expand upon the framework of logic in order to capture the real world. Here is an attempt to provide some clarity in relation to real world examples.

The essential point is that truth has a structure on the space of possible statements. Without such a structure logic does not describe the way real world truths are represented by human discourse and cognition.

The first simple example I want to discuss is a mathematical one, an oscillator (fig 1). The variable Y changes back and forth from positive to negative values over time. We can consider the statement "Y is positive" and realize that this statement is sometimes but not always true. We can write a statement that we can all agree on about the truth of this statement by saying that it is true at such and such times, and false at other times. Being able to say this, however, does not capture sufficiently the difficulty of logic in relation to the world.

We can imagine that the variable Y describes the emotional state of an individual. Due to internal dynamical processes of his/her endocrine system, the individual, without external changes of conditions, oscillates in feeling good and bad. The person, might then say "I feel good, I feel bad, I feel good, I feel bad." That statement in itself, is logically false. The missing information about the time at which each clause is stated is the problem. Still, a rational person would reasonably say that this person is "illogical" based upon the inconsistency (over time). And if the person were to say "It's a good day, it's a bad day, it's a good day, it's a bad day"—reflecting their dynamic emotional state in valuing conditions that themselves did not change—the objective of shared truths would manifestly fail even at the level of the individual agreeing with him or her self.

Dynamical behaviors of human emotions can be much more complicated than the simple oscillator of Fig. 1. They can combine external conditions, prior information, and internal dynamics that result in time delays in effect.

As in the above example about whether the day is a good one, the dynamics of human emotion are a central part of human understanding, shifting interpretation of events and focus of attention on partial information. Indeed, human cognition is a dynamical system, influenced by internal and external information, which is incomplete. These change the essential nature of what an individual considers to be truth and its communication with others. Logic is designed to avoid these complexities precisely because it does not seek to describe the subjectivity of human truth in its complexity and instead abstracts those statements that can be commonly agreed upon. These objectives, however, themselves cannot serve to understand collectivity of human communication both because of what is defined by individuals as truth, and because the same issues arise also in how and what collectivities agree upon is truth.

Formally: Temporal logic(s) describe how inference works when we say things like "I was hungry until I had something to eat." Modeling enables us to say that given a certain model of emotional variation, I will feel good 5 minutes after I feel bad. The recognition of relational properties also provides a context for subjective truth. Yet, and paradoxically, formalizing "I thought it was a good day before I thought it was not a good day," so that we can all agree on when or why a person feels the way they do, leaves us to grapple with the consequences of the failure of logic to achieve its purpose in reaching collective agreement.

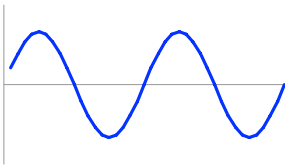

FIG. 1: Oscillator having alternating positive and negative values.